A Fully Smooth Neural Network for Nonlinear Semidefinite Programming via Smoothed Fischer–Burmeister Functions

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Abstract

We propose a fully smooth continuous-time neural network for solving convex nonlinear semidefinite programming (NSDP) problems with both matrix inequality and scalar inequality constraints. Both constraint types are handled through smoothed Fischer–Burmeister (FB) functions: the matrix-valued extension Φ_μ is used for the linear matrix inequality (LMI), and the scalar smooth FB function ϕ_μ is applied componentwise to the inequality constraints. The resulting energy function $E(y) = \frac{1}{2}\|\eta_\mu(y)\|^2$ is C^∞ for every $\mu > 0$, which yields a Lipschitz right-hand side and guarantees global existence and uniqueness of solution trajectories. We prove that every equilibrium of the network coincides with a KKT point of the NSDP and that the network is globally asymptotically stable. Numerical experiments on standard NSDP benchmark problems confirm that the proposed architecture converges from arbitrary initial conditions and achieves competitive performance against existing projection-based neural network methods.

Keywords: Fischer–Burmeister function, global stability, KKT conditions, neural network, nonlinear semidefinite programming, smoothing method, Lyapunov stability.